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Neville–Lagrange wavelet family for lossless image compression

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ABSTRACT

This paper presents a new wavelet family by combining the Neville filter theory and Lagrange interpolation. The filter banks of the new wavelet family are built and named as Neville–Lagrange lifting wavelet filter banks (N–LLWFBs for short). The prediction filters of N–LLWFBs are obtained by considering the signal sampling and Lagrange interpolation, and the corresponding update filters are given by using Neville filter theory. Examples are given by using this approach. The Neville–Lagrange prediction filters are obtained; causal lifting wavelet filter banks are also constructed by using this approach. Several N–LLWFBs for image compression are designed, and they are normalized in terms of the normalization conditions of the first generation wavelet filter bank. As a special example, the lifting scheme of 5/3 wavelet of JPEG2000 is obtained; it is the two-channel N–LLWFB of order 2 both dual and primal vanishing moments. Experiment results show that the performance of N–LLWFBs for image compression becomes better with the increase of their vanishing moments.

1. Introduction

Over the past decades, discrete wavelet transform and perfect reconstruction filter banks have become the dominant technologies in numerous areas such as signal and image processing [1–3]. Mallat [4] developed a pyramidal wavelet transform using a numerical filter bank. In recent years, the second-generation wavelets based on lifting scheme have achieved substantial recognition [5,6], especially since their integration in the JPEG2000 standard [7]. Lifting scheme is an efficient and powerful tool to compute the wavelet transform. It can improve the key properties of the first generation wavelet step by step. At the same time it has many advantages comparing with the first generation wavelet such as in-place computation, integer-to-integer transforms and speed. Daubechies and Sweldens [6] presented how to decompose biorthogonal wavelet filter banks into lifting steps.

The common design of lifting wavelet filter bank is to decompose the first generation wavelet filter into lifting steps. That is to say, this design is heavily dependent on the existed wavelets. Kovačević and Sweldens [8] constructed the wavelet families of increasing order in arbitrary dimensions. The theoretical basis of their method is the Neville filter theory, which is named by Kovačević and Sweldens and is strongly related with the interpolating filter bank. The mathematic basis of Kovačević and Sweldens' method is the de Boor–Ron algorithm [9,10]. Neville filter theory can give a perfect interpretation for the meanings of the prediction filter P and the update filter U. The most important advantage of Neville filter theory is that the lifting filters can be designed directly using Neville filter theory, not in depending on the existed wavelets.

Lagrange interpolation is a perfect common of multi-point prediction [11,12]. It can predict the value of a center location by using neighboring points. We find that the weights of the locations from the Lagrange interpolation formula can constitute the impulse response of the

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prediction filter P of the lifting wavelet filter bank. Therefore, we can combine Neville filter theory and Lagrange interpolation to construct the new lifting wavelet filter banks.

This work aims to design a new lifting wavelet family based on Neville filter theory and Lagrange interpolation. The prediction filters P_i $(1\leqslant i\leqslant M-1)$ of lifting wavelet are constructed using Lagrange interpolation and Neville filter theory. The corresponding update filter U_i $(1\leqslant i\leqslant M-1)$ can be obtained using Neville filter theory. Hence the lifting filter banks named as Neville–Lagrange lifting wavelet filter banks (N–LLWFBs) are built.

The remainder of the paper is organized as follows. Section 2 gives the brief description of the background of wavelet filter bank and the Neville filter theory. Section 3 gives the Lagrange interpolation formula on m/n-taps. Section 4 describes the relationship between prediction filter and down sampling, and the common equation of prediction filters in terms of down sampling. Section 5 introduces several examples and discusses the computational complexity of Neville–Lagrange lifting filter banks. In Section 5, the prediction filters of Neville–Lagrange Lifting wavelet family are built with $M=2$, 4; the causal M-channel N–LLWFBs are presented; moreover, the computational complexity of N–LLWFBs is discussed. Section 6 introduces the design of N–LLWFBs for lossless image compression. In Section 6, an N–L-22 lifting wavelet filter bank, which has order 2 both dual and primal vanishing moments, is constructed, and it is just the lifting scheme of 5/3 wavelet of JPEG2000. Furthermore, the normalization of N–L-22 is also discussed in terms of the normalization conditions of first generation wavelet filter bank. Section 7 presents the experiments of N–LLWFBs for image compression using the objective and subjective assessment. Conclusion is given in Section 8.

2. Wavelet filter bank and Neville filter theory

2.1. Three representations of wavelet filter bank

There are three representations on wavelet filter bank (see Fig. 1): channel representation, polyphase representation and lifting representation. The relationship of three representations is very important for constructing and analyzing wavelet filter bank. Here, the lifting wavelet filter bank with only one dual lifting step and one primal lifting step for the ith channel $(1\leqslant i\leqslant M-1)$ is considered; the relationship equations of the three representations of wavelet filter bank are obtained. Especially, the relationship equations between the lifting representation and the channel representation of wavelet filter bank are given.

In Fig. 1(a), the M-polyphase decompositions of $\tilde{G}_i(z^{-1})$ and $G_i(z)$, $0\leqslant i\leqslant M-1$ are given by

$$\tilde{G}_i(z^{-1})=\sum_{j=0}^{M-1} z^j\tilde{G}_{i,j}(z^{-M}) \tag{1}$$

$$G_i(z)=\sum_{j=0}^{M-1} z^{-j}G_{i,j}(z^{M}) \tag{2}$$

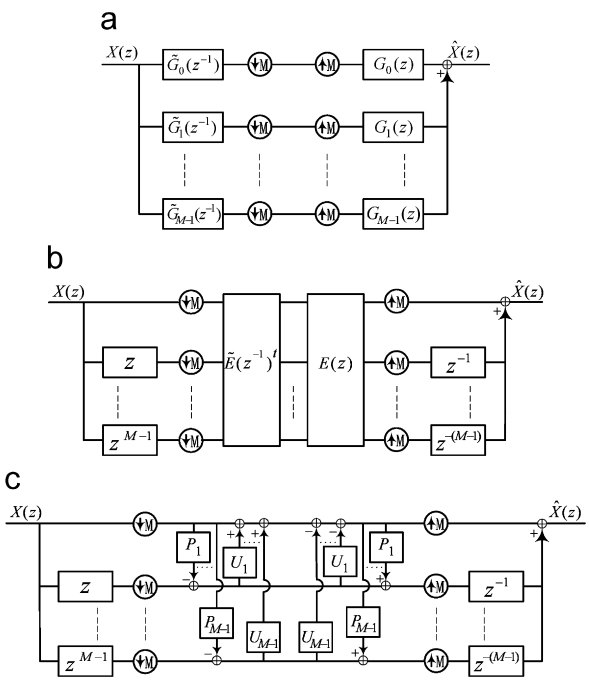

Fig. 1. Three representations of wavelet filter bank: (a) channel representation, (b) polyphase representation and (c) lifting representation.

where $\tilde{G}_0$, $\tilde{G}_i(1\leqslant i\leqslant M-1)$, G_0, $G_i(1\leqslant i\leqslant M-1)$ denote analysis low-pass filter, analysis high-pass filters, synthesis low-pass filter, and synthesis high-pass filters, respectively.

The polyphase representation and the lifting representation of wavelet filter bank are shown in Figs. 1(b) and (c), respectively. In this section, we employ two lifting steps for the ith channel $(1\leqslant i\leqslant M-1)$, one prediction filter (P_i) and one update filter (U_i) (see Fig. 1(c)). The relationship equations between polyphase representation and lifting representation are given by

$$\tilde{E}(z)=\begin{bmatrix} \tilde{G}_{0,0} & \tilde{G}_{1,0} & \tilde{G}_{2,0} & \cdots & \tilde{G}_{M-1,0} \\ \tilde{G}_{0,1} & \tilde{G}_{1,1} & \tilde{G}_{2,1} & \cdots & \tilde{G}_{M-1,1} \\ \tilde{G}_{0,2} & \tilde{G}_{1,2} & \tilde{G}_{2,2} & \cdots & \tilde{G}_{M-1,2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{G}_{0,M-1} & \tilde{G}_{1,M-1} & \tilde{G}_{2,M-1} & \cdots & \tilde{G}_{M-1,M-1} \end{bmatrix}$$

$$=\begin{bmatrix} 1-\sum_{j=1}^{M-1} U_j^*P_j^* & -P_1^* & -P_2^* & \cdots & -P_{M-1}^* \\ U_1^* & 1 & 0 & \cdots & 0 \\ U_2^* & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ U_{M-1}^* & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{3}$$

$$E(z)=\begin{bmatrix} G_{0,0} & G_{1,0} & G_{2,0} & \cdots & G_{M-1,0} \\ G_{0,1} & G_{1,1} & G_{2,1} & \cdots & G_{M-1,1} \\ G_{0,2} & G_{1,2} & G_{2,2} & \cdots & G_{M-1,2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ G_{0,M-1} & G_{1,M-1} & G_{2,M-1} & \cdots & G_{M-1,M-1} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -U_1 & -U_2 & \cdots & -U_{M-1} \\ P_1 & 1-U_1P_1 & -U_2P_1 & \cdots & -U_{M-1}P_1 \\ P_2 & -U_1P_2 & 1-U_2P_2 & \cdots & -U_{M-1}P_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{M-1} & -U_1P_{M-1} & -U_2P_{M-1} & \cdots & 1-U_{M-1}P_{M-1} \end{bmatrix} \tag{4}$$

where $\tilde{E}(z)$ and $E(z)$ are the polyphase matrices of polyphase representation. $P_i(1\leqslant i\leqslant M-1)$ and $U_i(1\leqslant i\leqslant M-1)$ are the prediction filters and the update filters of the lifting representation, respectively. Here, let $P_i^*(z)=P_i(z^{-1})$ and $U_i^*(z)=U_i(z^{-1})$, $1\leqslant i\leqslant M-1$.

Considering the above Eqs. (1)–(4), there are

$$\tilde{G}_0(z)=1+\sum_{j=1}^{M-1}(z^{-j}U_j^*(z^M)-P_j^*(z^M)U_j^*(z^M)) \tag{5}$$

$$\tilde{G}_i(z)=z^{-i}-P_i^*(z^M),\quad 1\leqslant i\leqslant M-1 \tag{6}$$

$$G_0(z)=1+\sum_{j=1}^{M-1}z^{-j}P_j(z^M) \tag{7}$$

$$G_i(z)=-U_i(z^M)+z^{-i}-\sum_{j=1}^{M-1}z^{-j}U_i(z^M)P_j(z^M),$$
$$1\leqslant i\leqslant M-1 \tag{8}$$

where $P_i^*(z)=P_i(z^{-1})$, $U_i^*(z)=U_i(z^{-1})$, and $P_i^*(z^M)=P_i(z^{-M})$, $U_i^*(z^M)=U_i(z^{-M})$.

Based on above four Eqs. (5)–(8), $\tilde{G}_0$, $\tilde{G}_i(1\leqslant i\leqslant M-1)$, G_0, $G_i(1\leqslant i\leqslant M-1)$ can be obtained in terms of $P_i(1\leqslant i\leqslant M-1)$ and $U_i(1\leqslant i\leqslant M-1)$.

2.2. Neville filter theory and lifting scheme

Kovačević and Sweldens built the wavelet families of increasing order in arbitrary dimensions. The main idea in their research is the Neville filter theory. The work is indeed a direct inspirational product of Kovačević and Sweldens's work in [8]. To build our latter Neville–Lagrange lifting wavelet filter bank, we simplified the definition and theorems of Neville filter theory from arbitrary dimensions to one dimension, described as Definition 1 (refer to Definition 3 in [8]), Theorem 1 (refer to Proposition 4 in [8]), and Theorem 2 (refer to Proposition 5 in [8]). The relation theorem between Neville filter and lifting is denoted as Theorem 3 (refer to Theorem 10 in [8]).

Definition 1. A filter P is a Neville filter of order N with shift $\tau\in\mathbf{R}$ if $P\pi(\mathbf{Z})=\pi(\mathbf{Z}+\tau)$ for $\pi\in\Pi_N$.

Here $\mathbf{Z}$ is integer set, π is a polynomial sequence, and Π_N denotes the space of all polynomial sequences of total degree strictly less than N.

In Definition 1, applying a Neville filter P to a polynomial sequence $\pi(\mathbf{Z})$ results in the polynomial sequence $\pi(\mathbf{Z}+\pi)$, which is the original sequence offset by τ. Therefore, A Neville filter P can be regard as a prediction filter, which is crucial for the construction of lifting filter banks.

Theorem 1. *A filter P is a Neville filter of order N with shift $\tau\in\boldsymbol{R}$ if and only if its impulse response satisfies $\sum_k p_{-k}k^n=\tau^n$, for $n<N$.*

Theorem 1 shows how to prove that a filter is a Neville filter of order N with shift τ using its impulse response.

Theorem 2. *If P is a Neville filter of order N with shift $\tau\in\boldsymbol{R}$, then $P^*(P^*(z)=P\ (z^{-1}))$ is a Neville filter of order N with shift $-\tau$.*

Theorem 3. *Let $N\leqslant\tilde{N}$. We can build an M-channel lifting wavelet filter bank with $\tilde{N}$ dual vanishing moments and N primal vanishing moments by letting the prediction filters $P_i(1\leqslant i\leqslant M-1)$ be Neville filters of order $\tilde{N}$ with shifts $\tau_i=i/M$ and choosing the update filters $U_i(1\leqslant i\leqslant M-1)$ as $1/M$ times of Neville filters of order N with shifts $-\tau_i$.*

Theorem 3 is the construction theorem of M-channel lifting wavelet filter bank with $\tilde{N}$ dual vanishing moments and N primal vanishing moments using Neville filter theory. In Theorem 3, the construction of Neville filters of order $\tilde{N}$ with shift $\tau_i=i/M$ is a core problem, and the prediction filters and update filters can be obtained using the Neville filters.

3. Lagrange interpolation on *m*/*n*-taps

M-channel wavelet transform based on the lifting scheme decomposes the signal into M disjoint sets using the lazy wavelet transform. One can predict the other $M-1$ sets by using the special one. Any point of the $M-1$ sets can be predicted using neighboring points of the special set. These points of the special set, which are closer to this point predicted, should have more weights, and the weights should decrease with the increasing distance. Lagrange interpolation is a common method of multipoint prediction, and is consistent with above rules.

Lagrange interpolation can be used to predict a certain point in terms of its neighboring points. In Fig. 2, we assume that the value of each interval is equal to na; the cross point between the horizontal axis and the left-hand dashed line is the point x that we want to predict; the

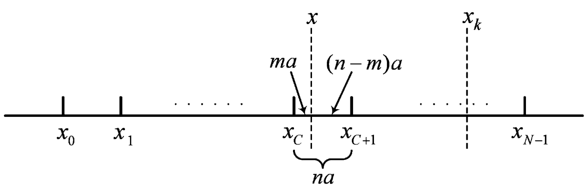

Fig. 2. Lagrange interpolation on m/n-taps.

cross point between the horizontal axis and the right-hand dashed line is one of the points x_k that will be used to predict. The point x divides the interval $[x_C, x_{C+1}]$ into two subintervals $[x_C, x]$ and $[x, x_{C+1}]$ with lengths of ma and $(n-m)a$, respectively. The goal using Lagrange interpolation is to obtain the weight of any point x_k when predicting the value of the point x.

Formula of Lagrange interpolation is given as follows:

$$P_{N,C}(x) = \sum_{k=0}^{N-1}\left(\frac{\prod_{j=0,j\neq k}^{N-1}(x-x_j)}{\prod_{j=0,j\neq k}^{N-1}(x_k-x_j)}\right)y_k = \sum_{k=0}^{N-1} L_{N,C,k}^{n,m} y_k \tag{9}$$

where N is the number of total points that will be used to interpolate, C is the subscript of the point x_C that is the left endpoint of interval $[x_C, x_{C+1}]$ including x, y_k is the value of signal at x_k, and $L_{N,C,k}^{n,m}$ is the weight at point x_k when predicting the value at point x.

In Fig. 2, we have

$$\begin{aligned}(x-x_0)(x-x_1)\cdots(x-x_{N-1}) &= (Cna+ma)((C-1)na+ma)\cdots(na+ma)ma(-(n-m)a)\\ &\quad\times[-na-(n-m)a]\cdots[-((N-1)-(C+1))na-(n-m)a]\\ &= (Cn+m)((C-1)n+m)\cdots(n+m)m\\ &\quad\times(n-m)(2n-m)\cdots((N-1-C)n-m)(-1)^{N-1-C}a^N\\ &= \prod_{j=0}^{C}(jn+m)\prod_{j=1}^{N-1-C}(jn-m)(-1)^{N-1-C}a^N\end{aligned} \tag{10}$$

Solving $(x-x_k)$, there is

$$x - x_k = ((C-k)n+m)a \tag{11}$$

Substitute (11) into (10), and eliminate $(x-x_k)$

$$\begin{aligned}\prod_{j=0,j\neq k}^{N-1}(x-x_j) &= \frac{(x-x_0)(x-x_1)\cdots(x-x_{N-1})}{x-x_k}\\ &= \frac{\prod_{j=0}^{C}(jn+m)\prod_{j=1}^{N-1-C}(jn-m)(-1)^{N-1-C}a^{N-1}}{(C-k)n+m}\end{aligned} \tag{12}$$

In Fig. 2, we also have

$$\begin{aligned}\prod_{j=0,j\neq k}^{N-1}(x_k-x_j) &= (x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_{N-1})\\ &= (kna)((k-1)na)\cdots na(-na)\cdots(-(N-1-k)na)\\ &= k(k-1)\cdots1\cdot1\cdot2\cdots(N-1-k)(-1)^{N-1-k}(na)^{N-1}\\ &= \prod_{j=1}^{k}j\prod_{j=1}^{N-1-k}j(-1)^{N-1-k}(na)^{N-1}\end{aligned} \tag{13}$$

Substituting (12) and (13) into ${L_{N,C,k}}^{n,m}$ of (9) leads to

$$\begin{aligned}L_{N,C,k}^{n,m} &= \frac{\prod(x-x_j)}{\prod(x_k-x_j)}, \quad 0\leqslant C,\ k\leqslant N-1\\ &= \frac{(-1)^{C-k}\prod_{j=0}^{C}(jn+m)\prod_{j=1}^{N-1-C}(jn-m)}{((C-k)n+m)n^{N-1}\prod_{j=1}^{k}j\prod_{j=1}^{N-1-k}j}\\ &\quad 0\leqslant C,\ k\leqslant N-1\end{aligned} \tag{14}$$

Here, $L_{N,C,k}^{n,m}$ is the weight of point x_k when predicting the value at point x.

Let $m/n = i/M (1\leqslant i\leqslant M-1)$, Eq. (14) can be rewritten as

$$\begin{aligned}L_{N,C,k}^{M,i} &= \frac{(-1)^{C-k}\prod_{j=0}^{C}(jM+i)\prod_{j=1}^{N-1-C}(jM-i)}{((C-k)M+i)M^{N-1}\prod_{j=1}^{k}j\prod_{j=1}^{N-1-k}j}\\ &\quad 0\leqslant C, k\leqslant N-1, \quad 1\leqslant i\leqslant M-1\end{aligned} \tag{15}$$

According to the latter Eq. (20), Theorems 5 and 6, we know that all values of $L_{N,C,k}^{M,i}$ in (15) with constant N, C, M and i constitute the coefficients of the prediction filter of the ith channel of Neville–Lagrange lifting wavelet filter bank.

Theorem 4. *Let* $1\leqslant i\leqslant M-1$, $0\leqslant C,k\leqslant N-1$, *if*

$$L_{N,C,k}^{M,i} = \frac{(-1)^{C-k}\prod_{j=0}^{C}(jM+i)\prod_{j=1}^{N-1-C}(jM-i)}{((C-k)M+i)M^{N-1}\prod_{j=1}^{k}j\prod_{j=1}^{N-1-k}j}$$

then we have $L_{N,C,k}^{M,i} = L_{N,N-2-C,N-1-k}^{M,M-i}$.

Proof. Considering $L_{N,C,k}^{M,i}$, we replace C, i, and k with $N-2-C$, $M-i$ and $N-1-k$, respectively,

$$\begin{aligned}&L_{N,N-2-C,N-1-k}^{M,M-i}\\ &= \frac{(-1)^{(N-2-C)(N-1-k)}\prod_{j=0}^{N-2-C}(jM+(M-i))\prod_{j=1}^{N-1-(N-2-C)}(jM-(M-i))}{(((N-2-C)-(N-1-k))M+(M-i))M^{N-1}\prod_{j=1}^{N-1-k}j\prod_{j=1}^{N-1-(N-1-k)}j}\\ &= \frac{(-1)^{-1-C+k}\prod_{j=0}^{N-2-C}((j+1)M-i)\prod_{j=1}^{1+C}((j-1)M+i)}{((-1-C+k)M+M-i)M^{N-1}\prod_{j=1}^{N-1-k}j\prod_{j=1}^{k}j}\\ &= \frac{(-1)^{C-k}\prod_{j=1}^{N-1-C}(jM-i)\prod_{j=0}^{C}(jM+i)}{((C-k)M+i)M^{N-1}\prod_{j=1}^{k}j\prod_{j=1}^{N-1-k}j}\end{aligned} \tag{16}$$

Comparing (15) and (16), there is $L_{N,C,k}^{M,i} = L_{N,N-2-C,N-1-k}^{M,M-i}$, and the proof is completed. □

Corollary 1. *Let* $1\leqslant i\leqslant M-1$, $0\leqslant C$, $k\leqslant N-1$, *if*

$$L_{N,C,k}^{M,i} = \frac{(-1)^{C-k}\prod_{j=0}^{C}(jM+i)\prod_{j=1}^{N-1-C}(jM-i)}{((C-k)M+i)M^{N-1}\prod_{j=1}^{k}j\prod_{j=1}^{N-1-k}j},$$

then we have $L_{N,(N/2)-1,k}^{2,1} = L_{N,(N/2)-1,N-1-k}^{2,1}$ *if N is an even number.*

Remark 1. Corollary 1 shows that the prediction filter of two-channel Neville–Lagrange lifting wavelet filter bank is obtained when letting $M=2$. Furthermore, let N is an even number and $C=(N/2)-1$, we can get the prediction filters of two-channel Neville–Lagrange filter bank with linear phase. Another point that we should note is that $\Pi_{j=a}^{b} f(j) = 1$ if $a>b$.

4. Prediction filter and down sampling

The Neville filter named by Kovačević and Sweldens is closely connected to the interpolating filter. The Neville filter theory is a common theoretical framework for the design of lifting wavelet filter banks with only one dual lifting step and one primal lifting step between low-pass and each high-pass channel. That is, any lifting wavelet filter bank with only one P and one U between low-pass and each high-pass channel must fit to the framework of Neville filter theory. Moreover, the Neville filter theory can give the perfect interpretation on the meanings of prediction filter P and update filter U.

In this section, the down sampling by M of original signal is considered. The prediction filter between low-pass and the ith high-pass channel is obtained by comparing the relationship of their sampling process.

According to Fig. 1(c), the down sampling of the low-pass channel is shown in Fig. 3.

In Fig. 3, the top figure denotes the original signal, and the sampling points of the low-pass channel are marked. The bottom figure in Fig. 3 denotes the down sampling by M of the original signal. The dashed line cross their center marks the center interval.

In Fig. 1(c), the sampling process of the ith high-pass channel is constituted by translation and down sampling. Hence it can be shown in Fig. 4.

In Fig. 4, the top figure shows the original signal, and the sampling points of the ith high-pass channel are marked. The middle figure is the left shift of the original signal. The bottom figure denotes the down sampling by M of the signal of the middle figure.

In Fig. 3, the point that needs to be predicted is z^{-CM-i}; this point is translated into z^{-C} according to the process of translation and decimation in Fig. 4. That is, to predict the point z^{-CM-i} by the sequence of points $(\ldots,z^0,z^{-M},\ldots,z^{-CM},z^{-CM-M},\ldots)$ in the top figure of Fig. 3, one can predict the point z^{-C} in the bottom figure of Fig. 4 by the sequence of points $(\ldots,z^0,z^{-1},\ldots,z^{-C},z^{-C-1},\ldots)$ in the bottom figure of Fig. 3.

Therefore, to predict the point z^{-C} of the ith high-pass channel (see the bottom figure in Fig. 4), the prediction filter that applied to the signal of low-pass channel (see the bottom figure in Fig. 3) must have the same symmetrical center, and the product among the corresponding points is z^{-C}. Therefore, the prediction filter can be shown in Fig. 5 as follows.

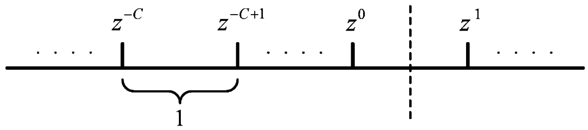

Fig. 5. Prediction filter of M-channel lifting wavelet filter bank.

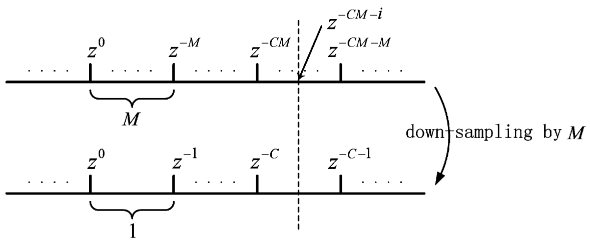

Fig. 3. Sampling of the low-pass channel in Fig. 1(c).

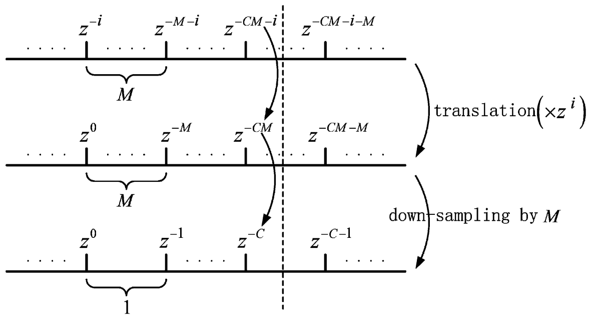

Fig. 4. Transaction and sampling of the ith high-pass channel in Fig. 1(c).

According to Fig. 5, the prediction filter $P(z)$ can be given by

$$P(z) = \cdots + p_C z^{-C} + p_{C-1} z^{1-C} + \cdots + p_1 z^{-1} + p_0 z^0 + p_{-1} z^1 + \cdots + p_{C-(N-1)} z^{N-1-C} + \cdots \quad (17)$$

Let the length of $P(z)$ is N, hence it can be rewritten as

$$P_{N,C}(z) = p_C z^{-C} + p_{C-1} z^{1-C} + \cdots + p_1 z^{-1} + p_0 z^0 + p_{-1} z^1 + \cdots + p_{C-(N-1)} z^{N-1-C} \quad (18)$$

That is,

$$P_{N,C}(z) = \sum_{k=0}^{N-1} p_{C-k} z^{k-C} \quad (19)$$

In (19), let $p_{C-k} = L_{N,C,k}^{M,i}$, and $P_{N,C}(z)$ can be rewritten as

$$P_{N,C}^{M,i}(z) = \sum_{k=0}^{N-1} p_{C-k} z^{k-C} = \sum_{k=0}^{N-1} L_{N,C,k}^{M,i} z^{k-C} \quad (20)$$

where $N \in \mathbf{Z}^{+}$, $0 \leqslant C \leqslant N-1$, $M \geqslant 2$ and $1 \leqslant i \leqslant M-1$.

In (15), let $i = 1,2,\ldots,M-1$, the filters $P_{N,C}^{M,i}(z)$ constitute the prediction filters of M-channel lifting wavelet filter bank. The design of the prediction filters is based on the coefficients formula $L_{N,C,k}^{M,i}$ of Lagrange interpolation, and it can be proven that the lifting wavelet filters designed in this way satisfy Theorem 1 (see Theorems 5 and 6). Hence we call the prediction filters as M-channel Neville–Lagrange lifting wavelet filters.

Considering (15) and (20), we have the following theorem:

Theorem 5. *Let* $1 \leqslant N \leqslant 10$, $2 \leqslant M \leqslant 10$, *we have* $\sum_{k=0}^{N-1} L_{N,C,k}^{M,i}(k-C)^L = (i/M)^L$ *where* $0 \leqslant L \leqslant N-1$ *and* $1 \leqslant i \leqslant M-1$.

Proof. The proof of Theorem 5 is very difficult if only using the symbol representation, however, it can be completed using computer program. Let N increase from 1 to 10; let M increase from 2 to 10; adjusting the values of C, i and L, Theorem 5 can be proven in terms of computer program and it can be described by the following pseudocode:

for $1 \leqslant N \leqslant 10$
  for $2 \leqslant M \leqslant 10$
    for $0 \leqslant C \leqslant N-1$
      for $1 \leqslant i \leqslant M-1$
        for $0 \leqslant L \leqslant N-1$
for $0 \leqslant k \leqslant N-1$
  compute $\sum_k L_{N,C,k}^{M,i}(k-C)^K$
compare $\sum_k L_{N,C,k}^{M,i}(k-C)^K$ with $(i/M)^L$ □

Remark 2. There are two computational rules in Theorem 5. One rule is $0^0 = 1$ that can be occurred when $k = C$ and

$L=0$; the other rule is that $\Pi_{j=a}^{b}f(j)=1$ if $a>b$, which appears in equation $L_{N,C,k}^{M,i}$.

Theorem 6. *Let* $1\leqslant N\leqslant 10$, $2\leqslant M\leqslant 10$, *if* $L_{N,C,k}^{M,i}$ *is presented as* (15), *then the prediction filters* $P_{N,C}^{M,i}(z)$ (*see* (20)) *are Neville filters of order N with shifts* $\tau=i/M$.

Proof. Combining Theorems 1 and 5, the proof can be completed. □

Remark 3. To simplify the proof, we limit the parameters N and M to intervals [1,10] and [2,10], respectively. Theorems 5 and 6 should be correct if the limited conditions on N and M are canceled, but the proof is need to be researched deeply.

Theorem 7. *Let* $N\leqslant\tilde{N}$, $1\leqslant N$, $\tilde{N}\leqslant 10$, $2\leqslant M\leqslant 10$, *if* $L_{N,C,k}^{M,i}$ *is presented as* (15), *then we can build an M-channel lifting wavelet filter bank with* $\tilde{N}$ *dual vanishing moments and N primal vanishing moments by letting the prediction filters be* $P_{N,C}^{M,i}(z), 1\leqslant i\leqslant M-1$ (*see* (20)) *and choosing the update filters* $U_i, 1\leqslant i\leqslant M-1$ *as* $(1/M)P_{\tilde{N},C}^{M,i}(z^{-1}), 1\leqslant i\leqslant M-1$.

Proof. Combining Theorems 6, 3 and 2, the proof can be completed. □

5. Examples of Neville–Lagrange lifting wavelet filter banks

5.1. Prediction filters of Neville–Lagrange lifting wavelet family

Based on above discussions, we can build the prediction filters of N–LLWFBs. In (15), for every fixed N, M and i, let C change from 0 to $N-1$, one can obtain N prediction filters, which can be presented in Eq. (20). It can be proved that these prediction filters corresponding to fixed N, M and i are Neville filters of order N with shifts $\tau=i/M$ using Theorems 5 and 6.

In (15), let $M=2$, There is

$$L_{N,C,k}^{2,1}=\frac{(-1)^{C-k}\prod_{j=0}^{C}(2j+1)\prod_{j=1}^{N-1-C}(2j-1)}{(2(C-k)+1)2^{N-1}\prod_{j=1}^{k}j\prod_{j=1}^{N-1-k}j}\quad 0\leqslant C,k\leqslant N-1 \tag{21}$$

Let $M=2$, Eq. (20) can be rewritten as

$$P_{N,C}^{2,1}(z)=\sum_{k=0}^{N-1}L_{N,C,k}^{2,1}z^{k-C} \tag{22}$$

Hence we can construct the prediction filters with shift $\tau=i/M=1/2$, shown in Table 1.

Remark 4. Table 1 shows the prediction filters of Neville–Lagrange filter banks when N is from 1 to 5. In Table 1, the causal Neville filters can be obtained when $C=N-1$, and the anticausal Neville filters are given when $C=0$. Let N be even and $C=(N/2)-1$, according to Corollary 1 and Remark 1, we can construct the most interesting prediction filters of two-channel Neville filters with linear phase.

Table 1
Neville–Lagrange prediction filters for orders 1–5

(N,C)	z^k									
	z^{-4}	z^{-3}	z^{-2}	z^{-1}	z^0	z^1	z^2	z^3	z^4	
(1,0)					1					/1
(2,0)					1	1				/2
(2,1)				−1	3					/2
(3,0)					3	6	−1			$/2^3$
(3,1)				−1	6	3				$/2^3$
(3,2)			3	−10	15					$/2^3$
(4,0)					5	15	−5	1		$/2^4$
(4,1)				−1	9	9	−1			$/2^4$
(4,2)			1	−5	15	5				$/2^4$
(4,3)		−5	21	−35	35					$/2^4$
(5,0)					35	140	−70	28	−5	$/2^7$
(5,1)				−5	60	90	−20	3		$/2^7$
(5,2)			3	−20	90	60	−5			$/2^7$
(5,3)		−5	28	−70	140	35				$/2^7$
(5,4)	35	−180	378	−420	315					$/2^7$

These filters are the Neville filters with shift 1/2.

In (15), let $M=4$, we have

$$L_{N,C,k}^{4,i}=\frac{(-1)^{C-k}\prod_{j=0}^{C}(4j+i)\prod_{j=1}^{N-1-C}(4j-i)}{(4(C-k)+i)4^{N-1}\prod_{j=1}^{k}j\prod_{j=1}^{N-1-k}j}\quad 0\leqslant C,k\leqslant N-1,\quad 1\leqslant i\leqslant 3 \tag{23}$$

Similarly, Eq. (20) can be rewritten as

$$P_{N,C}^{4,i}(z)=\sum_{k=0}^{N-1}L_{N,C,k}^{4,i}z^{k-C},\quad 1\leqslant i\leqslant 3 \tag{24}$$

Therefore, the prediction filters with shift $\tau=i/4$, $1\leqslant i\leqslant 3$ can be constructed, as shown in Table 2.

Table 2 shows the prediction filters of Neville–Lagrange filter banks when letting N be 2–4. Comparing Table 1 with Table 2, we find that Table 1 can be obtained by letting $i=2$ in Table 2. Table 1 describes the same Neville–Lagrange prediction filters with shift 1/2 as letting $i/M=2/4=1/2$ in Table 2.

5.2. Causal Neville–Lagrange lifting wavelet filter banks

In terms of the Lagrange interpolation equation (15) and the equation of prediction filter (20), it is possible to build causal N–LLWFBs by letting the prediction filter be a causal Neville filter and choosing the update filter in terms of the corresponding anticausal Neville filter.

In (15) and (20), let $N=\tilde{N}, C=\tilde{N}-1$, we can obtain causal prediction filters

$$P_{\tilde{N},\tilde{N}-1}^{M,i}(z)=\sum_{k=0}^{\tilde{N}-1}L_{\tilde{N},\tilde{N}-1,k}^{M,i}z^{k-(\tilde{N}-1)} \tag{25}$$

$$L_{\tilde{N},\tilde{N}-1,k}^{M,i}=\frac{(-1)^{\tilde{N}-1-k}\prod_{j=0}^{\tilde{N}-1}(jM+i)}{((\tilde{N}-1-k)M+i)M^{\tilde{N}-1}\prod_{j=1}^{k}j\prod_{j=1}^{\tilde{N}-1-k}j}\quad 1\leqslant i\leqslant M-1,\quad 0\leqslant k\leqslant\tilde{N}-1 \tag{26}$$

Table 2
Neville– Lagrange prediction filters for orders 2– 4

(N,C,i)	z^k							
	z^{-3}	z^{-2}	z^{-1}	z^0	z^1	z^2	z^3	
(2,0,1)				3	1			$/2^2$
(2,0,2)				1	1			$/2^1$
(2,0,3)				1	3			$/2^2$
(2,1,1)			−1	5				$/2^2$
(2,1,2)			−1	3				$/2^1$
(2,1,3)			−3	7				$/2^2$
(3,0,1)				21	14	−3		$/2^5$
(3,0,2)				3	6	−1		$/2^3$
(3,0,3)				5	30	−3		$/2^5$
(3,1,1)			−3	30	5			$/2^5$
(3,1,2)			−1	6	3			$/2^3$
(3,1,3)			−3	14	21			$/2^5$
(3,2,1)		5	−18	45				$/2^5$
(3,2,2)		3	−10	15				$/2^3$
(3,2,3)		21	−66	77				$/2^5$
(4,0,1)				77	77	−33	7	$/2^7$
(4,0,2)				5	15	−5	1	$/2^4$
(4,0,3)				15	135	−27	5	$/2^7$
(4,1,1)			−7	105	35	−5		$/2^7$
(4,1,2)			−1	9	9	−1		$/2^4$
(4,1,3)			−5	35	105	−7		$/2^7$
(4,2,1)		5	−27	135	15			$/2^7$
(4,2,2)		1	−5	15	5			$/2^4$
(4,2,3)		7	−33	77	77			$/2^7$
(4,3,1)	−15	65	−117	195				$/2^7$
(4,3,2)	−5	21	−35	35				$/2^4$
(4,3,3)	−77	315	−495	385				$/2^7$

These filters are the Neville filters with shift $i/4$.

In (15) and (20), let $C=0$, the anticausal prediction filters are given as follows:

$$P_{N,0}^{M,i}(z)=\sum_{k=0}^{N-1}L_{N,0,k}^{M,i}z^k \tag{27}$$

$$L_{N,0,k}^{M,i}=\frac{(-1)^k\prod_{j=1}^{N-1}(jM-i)}{(i-kM)M^{N-1}\prod_{j=1}^{k}j\prod_{j=1}^{N-1-k}j}\quad 1\leqslant i\leqslant M-1,\quad 0\leqslant k\leqslant N-1 \tag{28}$$

Considering Theorem 7 and (28), the causal update filter corresponding to $P_{N,0}^{M,i}(z)$ (see (27)) can be written as

$$U_{N,0}^{M,i}(z)=\frac{1}{M}P_{N,0}^{M,i}(z^{-1})=\frac{1}{M}\sum_{k=0}^{N-1}L_{N,0,k}^{M,i}z^{-k} \tag{29}$$

Therefore, let $N\leqslant\tilde{N}$, using Theorem 7, the causal N–LLWFBs with $\tilde{N}$ dual vanishing moments and N primal vanishing moments can be constructed. The prediction filter is given by composing (25) and (26), and the update filter can be obtained according to (28) and (29).

5.3. Computational complexity

In this section, we discuss the computational complexity of N–LLWFBs. The unit we use is the cost, measured in number of multiplications and additions, computed by using one sample pair $(s_l,d_{l,1},\ldots,d_{l,M-1})$, where $(s_l,d_{l,1},\ldots,d_{l,M-1})$ denotes the result of applying the down sampling to the original signal $x=\{x_l|l\in\mathbf{Z}\}$, s_l denotes the sampling of low-pass channel and $d_{l,i}(1\leqslant i\leqslant M-1)$ stands for the sampling of ith high-pass channel. The lengths of filters (prediction filter and update filter) of the Neville–Lagrange filter banks are equal to the orders of their vanishing moments, and the cost of applying a lifting wavelet filter bank pair (P_i,U_i), $1\leqslant i\leqslant M-1$ with $\tilde{N}$ dual vanishing moments and N primal vanishing moments can be calculated. The cost of applying a prediction filter P_i, $1\leqslant i\leqslant M-1$ with $\tilde{N}$ dual vanishing moments is $\tilde{N}$ multiplications and $\tilde{N}-1$ additions; the cost of applying an update filter U_i, $1\leqslant i\leqslant M-1$ with N primal vanishing moments is N multiplications and $N-1$ additions. Therefore, considering the channel number M, the total cost of analysis part is $2(M-1)(\tilde{N}+N-1)$.

For the two-channel Neville–Lagrange filter banks, the cost of analysis part is $2(\tilde{N}+N-1)$. For the case of the filter P and U are symmetric, and $\tilde{N}$ and N are even, the cost of applying the prediction P is $\tilde{N}/2$ multiplications and $\tilde{N}-1$ additions; the cost of applying the update U is $N/2$ multiplications and $N-1$ additions. Therefore, the total cost is $\frac{3}{2}(\tilde{N}+N)-2$.

6. Neville–Lagrange lifting wavelet filter banks for lossless image compression

In this section, two-channel N–LLWFBs with linear phase are built. Firstly, a two-channel Neville–Lagrange lifting wavelet filter bank of order 2 both dual and primal vanishing moments is constructed and normalized; it is just the lifting scheme of 5/3 wavelet of JPEG2000 standard. Furthermore, the prediction filters of Neville–Lagrange lifting wavelet family for image compression are described; they all have the property of linear phase. Finally, the experiments of N–LLWFBs for image compression are discussed.

6.1. Construction of N– L-22 lifting wavelet filter bank

In this section, a Neville–Lagrange lifting wavelet filter bank is constructed; it is two-channel lifting wavelet filter bank of order 2 for both dual and primal vanishing moments. Moreover, it is named as the Neville–Lagrange-22 (N–L-22 for short) lifting wavelet filter bank. Here, the number "2" denotes the order of vanishing moments.

In (15), let $N=2$, $C=(N/2)-1=0$, $M=2$, the Lagrange interpolation coefficients can be given by

$$L_{2,0,k}^{2,1}=\frac{(-1)^k}{(1-2k)2\prod_{j=1}^{k}j\prod_{j=1}^{1-k}j},\quad 0\leqslant k\leqslant N-1 \tag{30}$$

Hence

Let $k=0$, then $L_{2,0,0}^{2,1}=\frac{1}{2}$

Let $k=1$, then $L_{2,0,1}^{2,1}=\frac{1}{2}$

According to (20) the prediction filter of lifting wavelet filter bank can be written as

$$P(z)=\tfrac{1}{2}+\tfrac{1}{2}z \tag{31}$$

Considering Theorem 6, we have: $P(z)$ is a Neville filter of order 2 with shift 1/2.

The corresponding update filter $U(z)$ with vanishing moments of order 2 is given according to Theorem 7

$$U(z) = P(z^{-1})/2 = \tfrac{1}{4}z^{-1} + \tfrac{1}{4} \tag{32}$$

According to the next section, we will show that the prediction filter of the lifting scheme of 5/3 wavelet of JPEG2000 (JPEG2000-5/3 for short) is same to (31), and the update filter is just Eq. (32). Therefore, the lifting scheme of JPEG2000-5/3 has order 2 both dual and primal vanishing moments; and it can also be named as Neville–Lagrange-22 lifting wavelet filter bank.

6.2. Normalization of N– L-22 lifting wavelet filter bank

The coefficients of a wavelet filter bank must satisfy the normalization conditions of the first generation wavelet filter bank theory. Similarly, the lifting wavelet filter bank is also needed to satisfy normalization conditions. The difference between the first generation wavelet and the lifting the wavelet is the normalization mode. The normalization of the first generation wavelet is implemented by scaling the coefficients of the wavelet filter bank, but the normalization of the lifting wavelet is achieved by scaling the coefficients of a decomposed signal using lifting. In this section, the normalization method of the N–L-22 lifting wavelet filter bank is given.

Let $M = 2$, Eqs. (5)–(8) can be simplified as

$$\tilde{G}_0(z) = 1 + (z^{-1} - P^*(z^2))U^*(z^2) \tag{33}$$

$$\tilde{G}_1(z) = z^{-1} - P^*(z^2) \tag{34}$$

$$G_0(z) = 1 + z^{-1}P(z^2) \tag{35}$$

$$G_1(z) = z^{-1} - (1 + z^{-1}P(z^2))U(z^2) \tag{36}$$

where $\tilde{G}_0, \tilde{G}_1, G_0,$ and G_1 denote analysis low-pass filter, analysis high-pass filter, synthesis low-pass filter, and synthesis high-pass filter, respectively.

Considering multiresolution theory, it is known that the coefficients of the first generation wavelet filter bank, which satisfies biorthogonal conditions, must satisfy these conditions as follows:

$$\sum_k \tilde{g}_{0,k} = \sqrt{2}, \quad \sum_k \tilde{g}_{1,k} = 0, \quad \sum_k g_{0,k} = \sqrt{2}, \quad \sum_k g_{1,k} = 0 \tag{37}$$

For the N–L-22 lifting wavelet filter, $\tilde{G}_0, \tilde{G}_1, G_0, G_1$ can be obtained by combining (33)–(36), (31) and (32)

$$\begin{aligned}\tilde{G}_0(z) &= 1 + (z^{-1} - (\tfrac{1}{2} + \tfrac{1}{2}z^{-2}))(\tfrac{1}{4}z^2 + \tfrac{1}{4}) \\ &= -\tfrac{1}{8}z^{-2} + \tfrac{1}{4}z^{-1} + \tfrac{3}{4} + \tfrac{1}{4}z - \tfrac{1}{8}z^2\end{aligned} \tag{38}$$

$$\tilde{G}_1(z) = z^{-1} - (\tfrac{1}{2} + \tfrac{1}{2}z^{-2}) = -\tfrac{1}{2}z^{-2} + z^{-1} - \tfrac{1}{2} \tag{39}$$

$$G_0(z) = 1 + z^{-1}(\tfrac{1}{2} + \tfrac{1}{2}z^2) = \tfrac{1}{2}z^{-1} + 1 + \tfrac{1}{2}z \tag{40}$$

$$\begin{aligned}G_1(z) &= z^{-1} - (1 + z^{-1}(\tfrac{1}{2} + \tfrac{1}{2}z^2))(\tfrac{1}{4}z^{-2} + \tfrac{1}{4}) \\ &= -\tfrac{1}{8}z - \tfrac{1}{4} + \tfrac{3}{4}z^{-1} - \tfrac{1}{4}z^{-2} - \tfrac{1}{8}z^{-3}\end{aligned} \tag{41}$$

Therefore, according to Eqs. (38)–(41), we know that N–L-22 lifting wavelet filter bank is the lifting scheme of JPEG2000-5/3.

According to above four equations, we have

$$\sum_k \tilde{g}_{0,k} = 1, \quad \sum_k \tilde{g}_{1,k} = 0, \quad \sum_k g_{0,k} = 2, \quad \sum_k g_{1,k} = 0 \tag{42}$$

By comparing (37) and (42), the normalization is given as follows:

$$\begin{aligned}&\sum_k \tilde{g}_{0,k} = \sum_k \tilde{g}_{0,k}\sqrt{2}, \quad \sum_k \tilde{g}_{1,k} = \sum_k \tilde{g}_{1,k} \Big/ \sqrt{2}, \\ &\sum_k g_{0,k} = \sum_k g_{0,k} \Big/ \sqrt{2}, \quad \sum_k g_{1,k} = \sum_k g_{1,k}\sqrt{2}\end{aligned} \tag{43}$$

The above normalization is performed using the channel representation of the wavelet filter bank (see Fig. 1(a)). In this section, the normalization conditions are applied to the lifting scheme by stretching and shrinking the decomposition coefficients; it can be achieved by multiplying the scale factor $*\sqrt{2}, 1/\sqrt{2}, 1/\sqrt{2}, *\sqrt{2}$ to the low-pass analysis channel, the high-pass analysis channel, the low-pass synthesis channel, and the high-pass synthesis channel, respectively (see Fig. 6). After normalization, the following useful image compression system is obtained.

In Fig. 6, $\hat{X}(z)$ is the z-transform of input signal, $\hat{X}(z)$ is the z-transform of reconstruction signal. X_e and X_o denote the even sampling and the odd sampling of $X(z)$, respectively. P is the prediction filter and U is the update filter of lifting wavelet filter bank. $A(z)$ and $D(z)$ denote the approximate coefficients and the detail coefficients of wavelet transform, respectively.

6.3. Prediction filters of Neville– Lagrange lifting wavelet family for image compression

In above two sections, we discussed the construction and normalization of N–L-22 lifting wavelet filter bank. In this section, the other prediction filters of Neville–Lagrange lifting wavelet family for image compression are built. The prediction filters with more order of vanishing moments can be constructed by considering Eqs. (21) and (22). In Eqs. (21) and (22), let N be even and $C = (N/2) - 1$, according to Corollary 1 and Remark 1, we can construct the most interesting two-channel Neville filters with linear phase.

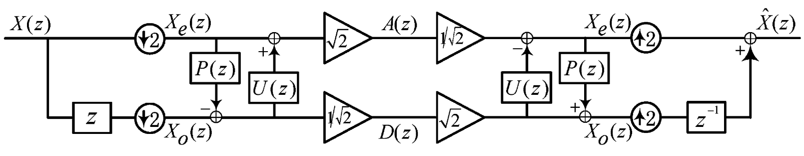

Fig. 6. Normalization of N–L-22 lifting wavelet.

Table 3
Prediction filters of Neville–Lagrange lifting wavelet family with linear phase

N	z^k								
	z^{-3}	z^{-2}	z^{-1}	z^0	z^1	z^2	z^3	z^4	
2				1	1				/2
4			−1	9	9	−1			$/2^4$
6		3	−25	150	150	−25	3		$/2^8$
8	−5	49	−245	1225	1225	−245	49	−5	$/2^{11}$

Let N be an even number and $C=(N/2)-1$, Eqs. (21) and (22) can be simplified as follows.

$$L^{2,1}_{N,(N/2)-1,k}=\frac{(-1)^{(N/2)-1-k}\prod_{j=0}^{(N/2)-1}(2j+1)\prod_{j=1}^{N/2}(2j-1)}{(N-1-2k)2^{N-1}\prod_{j=1}^{k}j\prod_{j=1}^{N-1-k}j}\quad 0\leqslant k\leqslant N-1 \tag{44}$$

$$P^{2,1}_{N,(N/2)-1}(z)=\sum_{k=0}^{N-1}L^{2,1}_{N,(N/2)-1,k}z^{k+1-(N/2)} \tag{45}$$

In (44) and (45), let N be 2, 4, 6, and 8, respectively, we obtain the prediction filters of Neville–Lagrange lifting wavelet family for image compression in Table 3 as follows.

In Table 3, the prediction filter of N–L-22 lifting wavelet filter bank constructed in Section 6.1 is obtained by letting N be 2; the order of vanishing moments of the prediction filter is 2. Similarly, the prediction filter with 4, 6 and 8 vanishing moments are constructed by letting N be 4, 6, and 8, respectively. Considering Theorem 7, the corresponding update filter with vanishing moments 4, 6 and 8 can been obtained. Therefore, we can build the N–LLWFBs with linear phase and with the same order both dual and primal vanishing moments.

According to Table 3, these lifting wavelet filter banks are named as Neville–Lagrange-44 (N–L-44 for short), Neville–Lagrange-66 (N–L-66 for short), and Neville–Lagrange-88 (N–L-88 for short), respectively. The normalizations of N–L-44, N–L-66 and N–L-88 are similar to the normalization of N–L-22; the same scale factors and the final diagram of image compression system which are identical with Fig. 6 can be obtained.

7. Experiments of N–LLWFBs for image compression

In this section, the objective and subjective measures for image compression using N–LLWFBs are discussed. The well-known PSNR is used in objective assessment, and the model of five-level rating scale is employed in subjective assessment. An image compression system (see Fig. 6) is constructed using the N–LLWFBs and is applied to the 512 × 512 8-bit gray-scale image Lena and texture image Barbara. The compression performance is compared among the N–L-22, N–L-44, N–L-66 and N–L-88. The SPIHT coding algorithm is used and the entropy coding is omitted.

7.1. Objective and subjective assessment for image compression

It is well known that the peak-signal-to-noise-ratio (PSNR for short) is a common objective criterion. The PSNR can be calculated using the following formula

$$\mathrm{PSNR}\triangleq 10\,lg\,\frac{(2^{255}-1)^2}{(1/N_1N_2)\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}(x[n_1,n_2]-\hat{x}[n_1,n_2])^2} \tag{46}$$

where $x[n_1,n_2]$ denotes the original image, $\hat{x}\,[n_1,n_2]$ stands for the reconstructed image, N_1 is the number of row, and N_2 is the number of column.

As the final users of images are humans, the subjective assessment for image compression by human observers is another import method except for the objective assessment. Both expert and non-expert observers are used in experiments; non-experts represent the average viewers while experts are believed to be able to give better, more 'refined' assessments of image quality since they have been trained and are familiar with images and their distortions. The rating scale used for the subjective evaluation is shown in Table 4.

The mean rating of a group of observers who join the evaluation is usually computed by

$$R=\left(\sum_{k=0}^{n}s_kn_k\right)\Big/\left(\sum_{k=0}^{n}n_k\right) \tag{47}$$

where s_k is the score corresponding to the kth rating, n_k denotes the number of observers with this rating, and n is the number of grades in the scale.

7.2. Objective and subjective assessment of N–LLWFBs for image compression

The compression experiments of smooth images Lena and texture image Barbara are given from Tables 5 to 8. Tables 5 and 6 show the objective assessment using PSNR; Tables 7 and 8 show the subjective assessment using rating scale. The experimental process of subjective assessment is described as follows.

Ten observers, chosen among people having different image processing backgrounds, are asked to subjectively evaluate the degraded images (Lena and Barbara). These degraded images are displayed on a high-resolution computer monitor, one at a time. The viewing distance is set to 60 cm. Each image occupies a square with a side length of 10 cm. The original image is always displayed in a fixed location on the screen. The scales for degraded

Table 4
Subjective rating scale

Degradation	Score
Imperceptible	1 (best)
Perceptible but not annoying	2
Slightly annoying	3
Annoying	4
Very annoying	5 (worst)

Table 5
PSNR of N-LLWFBs with different vanishing moments for image Lena

bpp	N–L-22	N–L-44	N–L-66	N–L-88
0.0625	28.388190	28.646888	28.668415	28.673161
0.125	31.085838	31.522480	31.585347	31.606539
0.25	34.013769	34.603189	34.716958	34.762549
0.375	35.806474	36.089086	36.274045	36.493304
0.5	37.347574	37.951237	38.004821	38.128203
0.75	39.090764	39.271681	39.517236	39.943360
1	40.825022	41.209175	41.376081	41.402028
2	45.097569	45.204475	45.530564	45.565976

Table 6
PSNR of N-LLWFBs with different vanishing moments for image Barbara

bpp	N–L-22	N–L-44	N–L-66	N–L-88
0.0625	22.761238	22.716716	22.673476	22.673006
0.125	23.703826	23.906033	23.982009	24.011992
0.25	25.770769	26.366012	26.572503	26.778435
0.375	27.809562	27.861638	28.242683	29.230180
0.5	29.321461	30.393102	30.818123	31.061414
0.75	31.725331	32.848914	33.161745	33.369653
1	34.144312	35.464328	35.942862	36.142676
2	41.127669	41.824135	42.009667	42.121895

Table 7
Subjective assessment of N-LLWFBs with different vanishing moments for image Lena

bpp	N–L-22	N–L-44	N–L-66	N–L-88
0.0625	5	5	5	5
0.125	5	4	4	4
0.25	3	3	3	3
0.375	3	2	3	2
0.5	2	2	2	2
0.75	1	1	1	1
1	1	1	1	1
2	1	1	1	1

Table 8
Subjective assessment of N-LLWFBs with different vanishing moments for image Barbara

bpp	N–L-22	N–L-44	N–L-66	N–L-88
0.0625	5	5	5	5
0.125	5	5	5	5
0.25	5	5	5	5
0.375	5	5	5	4
0.5	5	5	4	4
0.75	4	3	3	3
1	2	2	2	2
2	1	1	1	1

images (Lena and Barbara) are shown in Tables 7 and 8, respectively.

In Tables 5 and 6, we observe that the N–L-22 lifting wavelet filter bank, which is the lifting scheme of 5/3 wavelet of JPEG2000, has the worst performance than others, and the performance becomes better with the increase of vanishing moments.

Comparing the result in Table 5 with Table 7, Table 6 with Table 8, we can conclude that the objective assessment is consistent with the subjective assessment for the N–LLWFBs.

8. Conclusion

A new wavelet family based on Neville filter theory and Lagrange interpolation is constructed in this paper. The coefficients of prediction filters are calculated using Lagrange interpolation formula, and the corresponding update filters are obtained by using the Neville filter theory. The normalization conditions of N–LLWFBs are discussed. It is also proven that JPEG2000-5/3 is a special Neville–Lagrange lifting wavelet filter bank: Neville–Lagrange lifting wavelet filter bank with order 2 both dual and primal vanishing moments. The N–LLWFBs with more vanishing moments than N–L-22, such as N–L-44, N–L-66 or N–L-88, are applied to the image compression and they have better performance than N–L-22. Therefore, we consider that they have the potential ability to be the substitution of wavelet filter banks of JPEG2000 for lossless image compression.

We have to point out that the precondition of Theorem 5 is to limit the parameters N to interval [1,10] and M to interval [2,10], but this limit is not necessary. Further studies on Theorem 5 with arbitrary N and M will be considered in our future work.

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